$$\text{Nine days}=9\times24\times60\times60=777600 \text{ seconds}$$
$$\text{Deduct} \quad 475 \text{ seconds}$$

$\therefore$ Time of falling to surface of the earth$=777125$ seconds.

The corresponding distance is, $\frac{1}{2}gt^2=16\frac{1}{12}(777125)^2=9713099188802$ feet $=1839602119$ miles.

Also solved by *H. C. WHITAKER*, *G. B. M. ZERR*, *B. F. SINE*, *J. SCHEFFER*, *W. H. DRANE*, and *J. B. GREGG*. Mr. Gregg furnished a very elaborate solution, indicating the various steps of the computation, his result being 360733 miles. Dr. Whitaker gets the same result. The results of the various contributors differ widely, due to variously assumed values of the constants, and, in some cases, considering the earth a mere point.

95. Proposed by FLORIAN CAJORI, Ph. D., Author of History of Mathematics, History of Physics, etc., and Professor of Mathematics, Colorado College, Colorado Springs, Col.

Assuming that the velocity is proportionate to the distance described from the state of rest, (1) can the body start in motion? (2) If it can, what is its initial acceleration? If we make the additional assumption that the time of fall, from rest, through a finite distance is finite, does it follow that the distance is infinite?

Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

1. The statement here is somewhat confusing. No body can start in motion unless acted upon by an external force; if it be meant here then to ask, can the body be started, the answer is self-evident. Theoretically, an infinitesimal force would be sufficient to put the body in motion though the time might become infinite before the velocity became finite.

2. We have $ds/dt=ks$ where k is a constant depending upon initial conditions. Differentiating we get $d^2s/dt^2=k(ds/dt)=k^2s$; that is, the initial acceleration is k time the initial velocity and is itself proportional to the distance. What the value of this initial acceleration is, depends upon the external force acting and the mass of the body.

3. Nothing is said here about the initial force acting. If we assume it infinitesimal, it follows from 1, that if the time of fall through a finite distance is finite, then the velocity must be infinite.

DIOPHANTINE ANALYSIS.

78. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics in University of Tennessee, Knoxville, Tenn.

Find three square numbers in harmonical progression.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

The terms of an harmonical progression are the reciprocals of such numbers as form an arithmetical progression.

Let a^2, b^2, and c^2 be three square numbers in arithmetical progression, $a<b<c$. Then $b^2-a^2=c^2-b^2$, or $a^2+c^2=2b^2$. a, b, and c are rendered rational and integral by putting $a=p^2-q^2$ the difference between $2pq$, $b=p^2+q^2$, and $c=p^2-q^2+2pq$.

$\therefore$ The required harmonical progression is

$$1/(p^2-q^2-2pq)^2,\ 1/(p^2+q^2)^2,\ 1/(p^2-q^2+2pq)^2,$$

in which p and q are any integers, $p>q$.

Put $p=2$ and $q=1$. We then have $1/1^2$, $1/5^2$, $1/7^2$.

Put $p=3$ and $q=2$. We then have $1/7^2$, $1/13^2$, $1/17^2$.

II. Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

If a^2, b^2 are two numbers, then will a^2, $2a^2b^2/(a^2+b^2)$, b^2 be in harmonical progression. Let $a=m-n$, $b=m+n$; therefore, $(m-n)^2$, $(m^2-n^2)^2/(m^2+n^2)$, $(m+n)^2$ will be the required numbers, if $m^2+n^2=\square$.

Let $m=p^2-q^2$, $n=2pq$.

$\therefore$ $(q^2+2pq-p^2)^2$, $(p^4-6p^2q^2+q^4)^2/(p^2+q^2)^2$, $(p^2+2pq-q^2)^2$ are the numbers required.

Let $p=2$, $q=1$. $\therefore$ 1, $\frac{49}{25}$, 49 are the numbers.

Let $a=m$, $b=7m$. Then m^2, $49m^2/25$, $49m^2$ are the numbers required.

Let $m=5n$. $\therefore$ $25n^2$, $49n^2$, $1225n^2$ are the required numbers. This gives a series of whole numbers by giving n integral values.

III. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Me.; A. H. BELL, Hillsboro, Ill.; and CHAS. C. CROSS, Meredithville, Va.

As the reciprocal of numbers in arithmetical progression are in harmonical progression, we may solve the question by obtaining three squares in arithmetical progression.

Let $x^2-2xy+y^2$, x^2+y^2, and $x^2+2xy+y^2$, be the three squares, and it only remains to make $x^2+y^2=\square$. This is so when $x=p^2-q^2$ and $y=2pq$; then $x^2+y^2=(p^2+q^2)^2$ in which p and q may be any unequal numbers.

Take $p=2$, and $q=1$, and the squares are 1, 25, and 49, and the reciprocals are 1, $\frac{1}{25}$, and $\frac{1}{49}$, or if integrals are desired, 1225, 49, and 25.

Also solved by *O. S. WESTCOTT, SYLVESTER ROBINS, ALOIS F. KOVARIK, J. W. YOUNG*, and the *PROPOSER*.

79. Proposed by EDMUND FISH, Hillsboro, Ill.

Find an integral right triangle in which the bisector of one of the acute angles is also integral.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

In any right triangle, let a=altitude, b=base, c=hypotenuse, c_1=bisector of angle opposite base, c_2=bisector of angle opposite altitude.

Put $a=tpq$, $b=t(p^2-q^2)/2$, and $c=t(p^2+q^2)/2$; t, p, and q being any integers, $p>q$.

Then from solution I, problem 43, page 95, Vol. IV, No. 3, THE AMERICAN MATHEMATICAL MONTHLY, we find